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A REVIEW OF PLANING THEORY AND EXPERIMENT WITH A  
THEORETICAL STUDY OF PURE-PLANING LIFT OF  
RECTANGULAR FLAT PLATES

By Charles L. Shuford, Jr.

Langley Aeronautical Laboratory  
Langley Field, Va.



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## SUMMARY

A summary is given of the background and present status of the pure-planing flat-plate lift theories. The fundamental assumptions and the applicability to actual calculations of the planing lift force are reviewed.

A proposed theory based on the consideration of linear lifting-line theory less the suction component of lift plus crossflow effects is presented. A comparison of this theory with existing planing formulas and experimental data is made. The agreement between the results calculated by the proposed theory and the experimental data is satisfactory for engineering calculations of pure-planing rectangular-flat-plate lift and center of pressure.

## INTRODUCTION

Recent developments in water-based aircraft have resulted in configurations utilizing planing surfaces operating in ranges of trim, length-beam ratio, and Froude number beyond those for which most of the available planing theories were correlated with experimental data. In order to determine whether available planing theories are adequate in estimating the planing lift in these extended ranges, a review of these theories (refs. 1 to 13) and a correlation with existing data, including recent and unpublished data, were made and are presented herein. For purposes of expediency and simplification, this work is limited to the case of the rectangular flat plate in pure planing, that is, where buoyancy can be considered as negligible.

In addition to this review and correlation, an additional theory for the lift and center of pressure of a rectangular flat plate was developed and correlated with the pure-planing data. The proposed theory distinguishes between linear and nonlinear components of lift and is divided

into three parts: first, a reasonably accurate approximation to the linear components of lift; second, an estimation of the aerodynamic leading-edge-suction component of lift contained in the linear term; and third, a method for calculating the crossflow effects.

## SYMBOLS

A	aspect ratio, $b^2/S$
b	beam of planing surface, ft
$C_L$	lift coefficient, $\frac{L}{\frac{\rho}{2} V^2 S}$
$C_{LB}$	lift coefficient due to buoyancy, $\frac{L_B}{\frac{\rho}{2} V^2 S}$
$C_{Lb}$	lift coefficient based on square of beam, $\frac{\Delta}{\frac{\rho}{2} V^2 b^2}$
$C_{LS}$	lift coefficient based on principal wetted area, $\frac{\Delta}{\frac{\rho}{2} V^2 S} = \frac{C_{Lb}}{l_m/b}$
$C_N$	normal-force coefficient, $\frac{N}{\frac{\rho}{2} V^2 S}$
$C_V$	speed coefficient or Froude number, $V/\sqrt{gb}$
g	acceleration due to gravity, 32.2 ft/sec <sup>2</sup>
L	lift of planing surface, lb

$L_B$	lift due to buoyancy, lb
$l$	wetted length of planing surface or chord of airfoil, ft
$l^* = \frac{l}{b/2}$	
$l_m$	mean wetted length, ft
$l_p$	center-of-pressure location (measured forward of trailing edge), ft
$\left(\frac{l_p}{l_m}\right)_{calc}$	nondimensional center-of-pressure location
$m_o$	section lift-curve slope per radian
$N$	normal force, lb
$q$	free-stream dynamic pressure, $\frac{1}{2}\rho V^2$ , lb/sq ft
$S$	principal wetted area (bounded by trailing edge, chines, and heavy spray line), sq ft
$V$	horizontal velocity, fps
$w$	induced vertical velocity
$w_y$	induced vertical velocity at distance $y$ from center line of airfoil
$y$	distance from center line of airfoil to point where value of downwash is desired
$y^* = \frac{y}{b/2}$	
$\Gamma$	circulation or strength of vortex, $\frac{VlC_L}{2}$
$\gamma$	nondimensional loading parameter, $C_L l^*$

$\Delta$	vertical load, lb
$\eta$	distance from center line of airfoil to vortex
$\eta^* = \frac{\eta}{b/2}$	
$\theta = \cos^{-1} \eta^*$	
$\theta_1 = \cos^{-1} y^*$	
$\rho$	mass density of water, slugs/cu ft
$\tau$	trim (angle between planing bottom and horizontal), radians unless otherwise stated
$\tau_i$	induced angle of trim, $w/V$
$\tau_{iy}$	induced angle of trim at distance $y$ from center line of airfoil, $w_y/V$
$\phi$	Pabst's aspect-ratio correction factor based on the ratio of wetted length to mean beam

#### REVIEW OF EXISTING PLANING-LIFT THEORY

Wagner (ref. 1) considered the planing-force problem theoretically; however, his work is valuable mainly for the basic concepts presented in the application of the methods of airfoil theory to the planing problem. Wagner's work consists of studies of the flow processes and solutions for the force on an idealized two-dimensional planing surface; therefore, his work is not directly applicable for calculating the lift on a finite-aspect-ratio planing surface.

In planing theories such as that of Mayo (ref. 2) developed from virtual-mass considerations based on transverse flow, the assumption is made that the planing force can be calculated from the rate at which momentum is imparted to the downwash; however, the effect of aspect ratio is approximated by the Pabst empirical aspect-ratio correction factor (ref. 14).

In reference 3 Sokolov presented a combined theoretical and experimental solution of the planing problem. The theoretical formulas, which were developed for the two-dimensional case, were derived by using Bernoulli's equation and disturbance velocities. A finite-aspect-ratio planing-lift formula was developed by using Sottorf's experimental results (ref. 15) to determine empirically the value of the factor  $\epsilon$  which is the ratio of the change in velocity along the planing surface to the velocity of the free stream. The planing formula gives the lift forces in three components: the hydrostatic, the one due to circulation, and the one due to form. The solution given by Sokolov for pure-planing flat-plate lift is

$$C_{LS} = \epsilon(2 - \epsilon)\cos \tau \quad (1)$$

where curves for  $\epsilon$  are given in reference 3.

Sokolov gives a qualitative picture of the planing problem and determines the nature of the forces involved. The concepts presented, however, have not been used in the development of subsequent planing formulas, which have been empirical or follow the work of Wagner.

Perring and Johnston (ref. 4) presented the empirical relationship

$$C_{LS} = CA^n\tau \quad (2)$$

and by analyzing Sottorf's data (ref. 15) found the following formula to apply:

$$C_{LS} = 0.90A^{0.42}\tau \quad (3)$$

In reference 5, Sottorf proposed the formula

$$C_{LS} = 0.845A^{0.5}\tau \quad (4)$$

An equation that has a form similar to airfoil lifting-line theory was presented by Perelmutter (ref. 6). The equation is

$$C_{LS} = \frac{2A\tau}{1 + A} \quad (5)$$

Sedov (ref. 7) gives an equation based on the data of Sottorf (ref. 15) and Sambraus (ref. 16) which has the form

$$C_{LS} = \frac{0.7\pi A\tau}{A + 1.4} \quad (6)$$

An equation that contains a linear and nonlinear term was presented by Siler in reference 8. The linear term was obtained by assuming a form similar to airfoil lifting-line theory; however, the aspect-ratio factor was altered to give a deflected mass one-half that predicted by Jones (ref. 17) for a zero-aspect-ratio wing. The nonlinear term was obtained by a consideration of the transverse component of the flow (see ref. 18). The equation can be written in the form

$$C_{LS} = \frac{\pi A \sin \tau \cos \tau}{A + 4} + 0.88 \sin^2 \tau \cos \tau \quad (7)$$

In reference 9, Korvin-Kroukovsky, Savitsky, and Lehman proposed an equation derived primarily on the basis of the data of Sottorf (ref. 15) and Sambraus (ref. 16). The formula can be written as

$$C_{LS} = 0.012A^{0.5}(57.3\tau)^{1.1} \quad (8)$$

In reference 10, Korvin-Kroukovsky presented an equation that consisted of linear and nonlinear components. The linear term was obtained by a consideration of the downwash and the analytical solution for the potential flow about a planing surface developed by Wagner (ref. 1) and presented in detail by Pierson and Leshnover (ref. 19). The nonlinear term was obtained by a consideration of the transverse component of the flow. The equation has the form

$$C_{LS} = \frac{\pi A\tau}{A + 2} + 0.88\tau^2$$

However, this equation was empirically corrected to get better agreement with experimental data, so that approximately

$$C_{LS} = \frac{0.73\pi A\tau}{A + 2} + 0.88\tau^2 \quad (9)$$

Locke (ref. 11) assumed that the lift characteristics of low-aspect-ratio surfaces can be represented by a simple power function of the form

$$C_{LS} = DK\tau^n$$

where  $K$  and  $n$  depend only on aspect ratio and  $D$  is primarily a function of the operating conditions. For the case of the flat-plate planing surface Locke gives the equation

$$C_{LS} = 0.5K\tau^n \quad (10)$$

where curves for  $K$  and  $n$  are given in reference 11.

In reference 12 Perry assumed an equation for the ratio of planing lift to aerodynamic lift which converged to limits obtained by applying airfoil methods to the planing surface. The equation has the form

$$C_{LS} = MC_{L_{\text{airfoil}}} \quad (11)$$

where  $M$  represents the assumed equation for the ratio of planing lift to aerodynamic lift given by

$$M = \frac{\mu \frac{A}{2} + \frac{2}{\pi + 4} \sin \tau}{\frac{A}{2} + \frac{2}{\pi} \sin \tau}$$

and

$$\mu = \frac{\cos \tau}{1 + \cos \tau - (1 - \cos \tau) \log_e \left( \frac{1 - \cos \tau}{2 \cos \tau} \right) + \pi \sin \tau}$$

Curves for  $M$  and  $\mu$  are given in reference 12.

The limit of  $M$  for zero aspect ratio is 0.88 and for infinite aspect ratio is the value  $\mu$ . The value of  $M$  for zero aspect ratio is a result given by Bollay (ref. 18). The value of  $M$  for infinite aspect



ratio was obtained by a consideration of the analytical solution for the potential flow about a planing surface developed by Wagner (ref. 1) and presented in detail by Pierson and Leshnover (ref. 19).

An equation having a linear term with a form analogous to airfoil lifting-surface theory was proposed by P. R. Crewe of Saunders-Roe Ltd. (British) in correspondence between himself and the Langley Laboratory. This equation, based on the data of Kapryan and Weinstein (ref. 20), is

$$C_{LS} = \sin \tau \cos \tau \left[ \frac{8}{\pi} \frac{1}{1 + \sqrt{1 + \left(\frac{2}{A}\right)^2}} + 2 \sin \tau - B \sin^2 \tau \right] \quad (12)$$

where

$$B = 2.67 \quad (A < 2.0)$$

$$B = 3.0 \quad (A > 2.0)$$

Schnitzer (ref. 13) presented an equation derived from a consideration of two-dimensional deflected mass, modified for three-dimensional flow by the Pabst empirical aspect-ratio correction factor (ref. 14). The equation can be written in the form

$$C_{LS} = \phi \left( \frac{\pi^3 A}{16} \sin \tau \cos \tau + 0.88 \sin^2 \tau \right) \quad (13)$$

#### PROPOSED THEORY

An examination of experimental data indicates a pronounced nonlinear relationship between the planing lift coefficients and the angle of attack; therefore, linear theory would not provide adequate approximations to the planing lift. The determination of linear and nonlinear components of lift is the approach generally used in low-aspect-ratio airfoil theory. The present approach is based on the consideration of linear lifting-line theory less the suction component of lift plus viscous crossflow effects.

## Lift

Linear term.— The linear term is determined from a consideration of lifting-line airfoil theory. Since the heavy spray line (leading edge) of a planing surface is approximately elliptic, the airfoil theory is presented for an elliptic surface with elliptic loading and then modified for the planing case.

By use of the Prandtl airfoil theory, the airfoil lift coefficient is

$$C_L = m_0(\tau - \tau_i) \quad (14)$$

where  $m_0$  is the slope of the section lift curve.

If a sheet of trailing vortices located at 0.75 of the chord measured from the trailing edge and extending to infinity behind the airfoil is assumed, then from vortex theory the induced angle of attack is

$$\tau_{iy} = \frac{w_y}{V} = \frac{1}{4\pi V} \int_{-b/2}^{b/2} \frac{-d\Gamma \, d\eta}{d\eta (\eta - y)} \quad (15)$$

where  $\eta$  is the distance from the center line of the airfoil to the vortex and  $y$  is the distance from the center line of the airfoil to the point where the value of downwash is desired.

Now let

$$y^* = \frac{y}{b/2} = \cos \theta_1 \quad (16)$$

$$\eta^* = \frac{\eta}{b/2} = \cos \theta \quad (17)$$

$$l^* = \frac{l}{b/2} \quad (18)$$

and

$$\gamma = \frac{4\Gamma}{bV} \quad (19)$$

where

$$\gamma = C_L l^* = m_0 l^* (\tau - \tau_1) \quad (20)$$

From equation (15),

$$\tau_{1y} = \frac{1}{8\pi} \int_0^\pi \frac{d\gamma}{d\theta} \frac{d\theta}{\cos \theta - \cos \theta_1}$$

Now let

$$\gamma = \sum_{n=1}^{\infty} a_n \sin n\theta \quad (21)$$

then

$$\tau_{1y} = \frac{1}{8\pi} \int_0^\pi \frac{\sum_{n=1}^{\infty} n a_n \cos n\theta \, d\theta}{\cos \theta - \cos \theta_1} \quad (22)$$

The solution of this equation can be obtained by means of a recurrence formula and the solution of a linear finite-difference equation (ref. 21); thus,

$$\tau_{1y} = \sum_{n=1}^{\infty} \frac{n a_n}{8} \frac{\sin n\theta_1}{\sin \theta_1} \quad (23)$$

Since this equation is valid for any value of  $\theta_1$ , the subscripts on  $\tau_1$  and  $\theta$  can be dropped and equation (20) becomes

$$\tau = \sum_{n=1}^{\infty} \left( \frac{1}{m_0 l^*} + \frac{n}{8 \sin \theta} \right) a_n \sin n\theta \quad (24)$$

For an elliptic airfoil,

$$z^* = \frac{8}{\pi A} \sin \theta \quad (25)$$

and

$$\tau \sin \theta = \sum_{n=1}^{\infty} \frac{1}{8} \left[ \left( \frac{\pi A}{m_0} + n \right) \sin n\theta \right] a_n$$

Let

$$\tau \sin \theta = \sum_{n=1}^{\infty} b_n \sin n\theta \quad (26)$$

then

$$a_n = \frac{8b_n}{\frac{\pi A}{m_0} + n} \quad (27)$$

where

$$b_n = \frac{2}{\pi} \int_0^{\pi} (\tau \sin \theta) \sin n\theta \, d\theta \quad (28)$$

Let  $\tau$  be constant along the wing; then,

$$\begin{aligned} b_1 &= \tau & b_2 &= b_3 = b_{n \neq 1} = 0 \\ a_1 &= a_1 & a_2 &= a_3 = a_{n \neq 1} = 0 \end{aligned}$$

The lift coefficient is given by

$$C_L = \int_{-b/2}^{b/2} \frac{\rho V \Gamma}{q S} \, d\eta \quad (29)$$

By use of equations (17), (19), and (21), equation (29) becomes

$$C_L = \frac{A}{4} \int_0^\pi \gamma \sin \theta \, d\theta = \frac{\pi}{8} A a_n \quad (30)$$

Therefore, from equations (27) and (28),

$$C_L = \frac{\pi A \tau}{\frac{\pi A}{m_0} + 1} \quad (31)$$

which is the equation for the lift on an airfoil.

For a low-aspect-ratio planing surface having flow only on one side, the lift coefficient is assumed to be one-half the value given by equation (31) for a flat-plate airfoil, and  $m_0$ , the lift-curve slope for the two-dimensional planing surface, is assumed to be one-half the value given for a flat-plate airfoil; thus,

$$C_L = \frac{0.5\pi A \tau}{A + 1} \quad (32)$$

which gives the linear component of lift on a pure-planing flat plate.

Suction component of lift.— An airfoil has a suction component of lift due to the large negative pressures at the leading edge of the airfoil; however, for a planing surface this suction component of lift does not appear. Therefore, the lift obtained from the linear term (eq. (32)) is less by an amount equal to the suction component of lift given by

$$C_L = C_{L_{\text{linear}}} \sin^2 \tau \quad (33)$$

which is the value indicated by Wagner in reference 22.

The linear term (eq. (32)) less the suction component of lift is

$$C_L = \frac{0.5\pi A \tau}{1 + A} (1 - \sin^2 \tau) \quad (34)$$

Crossflow term.— For a simple theoretical consideration of the non-linear term, the velocity component perpendicular to the chord is assumed to be of the magnitude  $V \sin \tau$ . The drag coefficient for a planing surface of infinite aspect ratio is assumed to be 1.0, which is one-half the value given for a two-dimensional flat-plate airfoil. For the planing surface the flow is projected into components perpendicular and parallel to the planing-surface chord line, and the drag force associated with the flow perpendicular to the chord is calculated. Therefore, the normal force is

$$N = 1.0 \frac{\rho}{2} S(V \sin \tau)^2$$

and

$$C_N = \sin^2 \tau$$

or

$$C_L^* = \sin^2 \tau \cos \tau \quad (35)$$

which is a lift due to crossflow effects and is proportional to  $\sin^2 \tau$ , which is the concept presented for airfoils by Betz in reference 23.

Total lift.— The total lift on a pure-planing rectangular flat plate can be obtained by adding equations (34) and (35) and is

$$C_{LS} = \frac{0.5\pi A\tau}{1+A} (1 - \sin^2 \tau) + \sin^2 \tau \cos \tau \quad (36)$$

which represents the linear term less the suction component of lift plus the crossflow term. The magnitude of the crossflow effects, total lift, and total lift excluding suction effects is shown in figure 1.

Comparison of proposed and previous planing formulas.— A comparison of the proposed theory with previous planing formulas for constant length-beam ratios is given in figure 2. In figure 2(a) the proposed theory is compared with the planing formulas as presented by Sokolov (eq. (1)), Perring and Johnston (eq. (3)), Sottorf (eq. (4)), Perelmuter (eq. (5)), and Sedov (eq. (6)). In figure 2(b) the proposed theory is compared with the planing formulas presented by Siler (eq. (7)), Korvin-Kroukovsky (eq. (9)), and Schnitzer (eq. (13)). In figure 2(c) the proposed theory

is compared with the planing formulas presented by Korvin-Kroukovsky, Savitsky, and Lehman (eq. (8)), Locke (eq. (10)), and Crewe (eq. (12)).

The values given by the formula presented by Perry (ref. 12) were not plotted since the results depended on the airfoil data used. Perry showed that by using Winter's airfoil data (ref. 24) his formula approximated the results given by the formula presented by Korvin-Kroukovsky, Savitsky, and Lehman (ref. 9) for trims up to  $12^\circ$  and length-beam ratios below approximately 1.0.

#### Center of Pressure

The center of pressure on a planing surface of small aspect ratio may be considered to have two components, the component due to the linear-lift term less the suction effects and the component due to the crossflow term. The center of pressure for the lift due to the linear component of lift less the suction effects (eq. (34)) is assumed to be located at 0.75 of the mean wetted length from the trailing edge of the planing surface. The center of pressure for the lift due to the crossflow term (eq. (35)) is assumed to be located at the center of the mean wetted length; therefore,

$$\left(\frac{l_p}{l_m}\right)_{\text{calc}} = \frac{0.75(C_{L_{\text{total}}} - C_{L_{\text{crossflow}}}) + 0.5C_{L_{\text{crossflow}}}}{C_{L_{\text{total}}}} \quad (37)$$

which is a formula analogous to that used in airfoil theory. The components of lift are determined from equations (35) and (36).

#### COMPARISON OF THEORY AND EXPERIMENT

##### Buoyancy

The experimental data were considered as pure planing if the lift coefficient due to buoyancy, calculated from the wedge-shaped volumetric displacement of the planing surface below the level water surface and given by

$$C_{LB} = \frac{l_m}{b} \frac{1}{2C_v^2} \sin 2\tau \quad (38)$$

did not exceed a given value. The allowable lift coefficient due to buoyancy, as determined from equation (38), was arbitrarily selected as 0.01 at a trim of  $16^\circ$ . The allowable lift coefficient due to buoyancy for other trims was determined by drawing a straight line from zero trim and zero lift coefficient due to buoyancy through the value 0.01 at a trim of  $16^\circ$ . The permissible lift coefficient due to buoyancy for the data selected by this method at a trim of  $2^\circ$  varied from 16 percent at a length-beam ratio of 8 to 3.3 percent at a length-beam ratio of one-half. These values decreased with increasing trim so that at  $30^\circ$  they would vary from 6.5 percent at a length-beam ratio of 8 to 3.1 percent at a length-beam ratio of one-half. Analysis of unpublished Langley tank no. 2 data showed that subtraction of the lift coefficient due to buoyancy computed from equation (38) caused the data to approximately coincide (or collapse) for different speeds at low trims ( $4^\circ$ ); however, at the higher trims ( $16^\circ$ ) only approximately one-half of this value was needed to collapse the data.

### Lift

A comparison of the proposed theory with the experimental data of Weinstein and Kapryan (ref. 25), unpublished NACA data, data of Shoemaker (ref. 26), data of Locke (ref. 27), data of Sambraus (ref. 16), and data of Sottorf (ref. 15) is presented in figures 3 to 10. Only the experimental data indicated as pure planing by the method discussed in the preceding section have been considered.

Figure 3 gives a comparison of the proposed theory with the data of Weinstein and Kapryan (ref. 25). Figure 4 gives a comparison of the proposed theory with unpublished Langley tank no. 2 data. The data of Weinstein and Kapryan were obtained for a 4-inch-beam model tested at various loads and speeds and the unpublished data were from a 2.5-inch-beam model tested at a constant speed of 30 feet per second.

In figures 5 to 10 a comparison of experimental lift coefficients given in references 25, 26, 27, 16, and 15 and unpublished NACA data with the proposed formulas given by Crewe (eq. (12)), Locke (eq. (10)), Korvin-Kroukovsky, Savitsky, and Lehman (eq. (8)), and the present paper is presented. In general, the proposed theory gives an average of these data. The formula presented by Crewe (eq. (12)) is in good agreement, except at a trim of  $30^\circ$ , with the data of Weinstein and Kapryan (figs. 5(a) and 5(b)), with the data of Sambraus (fig. 9), and with the data of Sottorf (fig. 10). The formulas presented by Locke (eq. (10)) and Korvin-Kroukovsky, Savitsky, and Lehman (eq. (8)) do not give so good a representation of experimental data as the proposed theory or the formulas presented by Crewe (eq. (12)). There are no experimental data at high trims and large length-beam ratios to determine whether the planing formulas give the correct variation of lift in this region. The agreement



between the proposed theory and experiment is apparently satisfactory for engineering calculations of pure-planing rectangular-flat-plate lift in the ranges where experimental data are available.

#### Center of Pressure

The variation of center-of-pressure ratio with mean wetted-length—beam ratio for the data of Weinstein and Kapryan (ref. 25) is shown in figure 11 and for the unpublished Langley tank no. 2 data in figure 12. The proposed theory is shown by the solid line in figures 11 and 12 where

$$\frac{l_p}{b} = \left( \frac{l_p}{l_m} \right)_{\text{calc}} \frac{l_m}{b}. \quad \text{The agreement between the curve for the proposed}$$

theory and the experimental points appears to be satisfactory for engineering calculations of pure-planing rectangular-flat-plate center of pressure in the ranges where experimental data are available.

#### CONCLUDING REMARKS

The proposed theory appears to predict the pure-planing rectangular-flat-plate lift and center of pressure with engineering accuracy in the ranges where experimental data are available; however, at high trims and large wetted-length—beam ratios no data are available. The correlation of experimental data and theory in this report seems to establish firmly the utility of the  $\sin^2\tau$  approach (where  $\tau$  is trim) to the nonlinearity problem.

Langley Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
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27. Locke, F. W. S., Jr.: Tests of a Flat Bottom Planing Surface To Determine the Inception of Planing. NAVAER DR Rep. 1096, Bur. Aero., Dec. 1948.

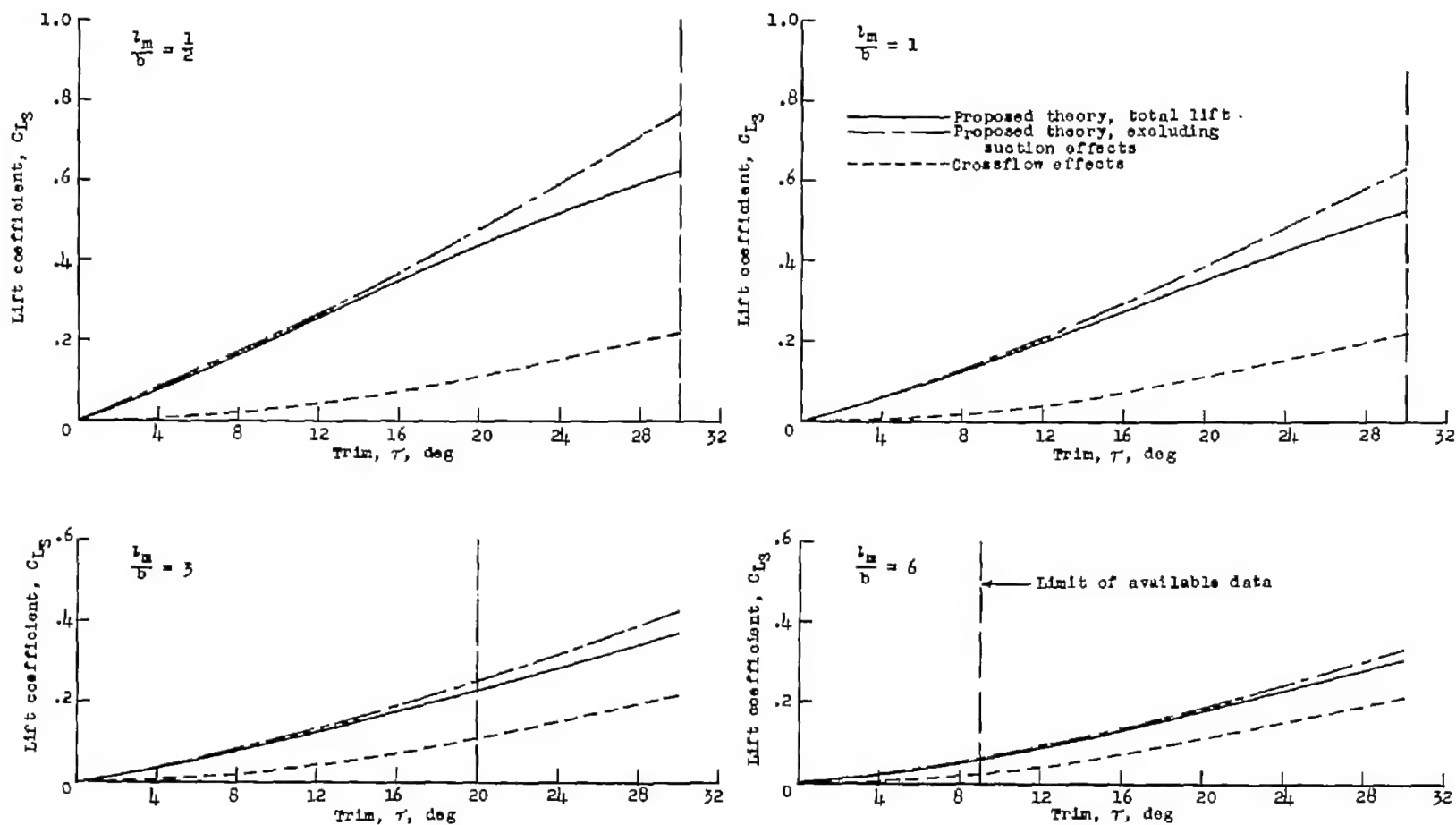
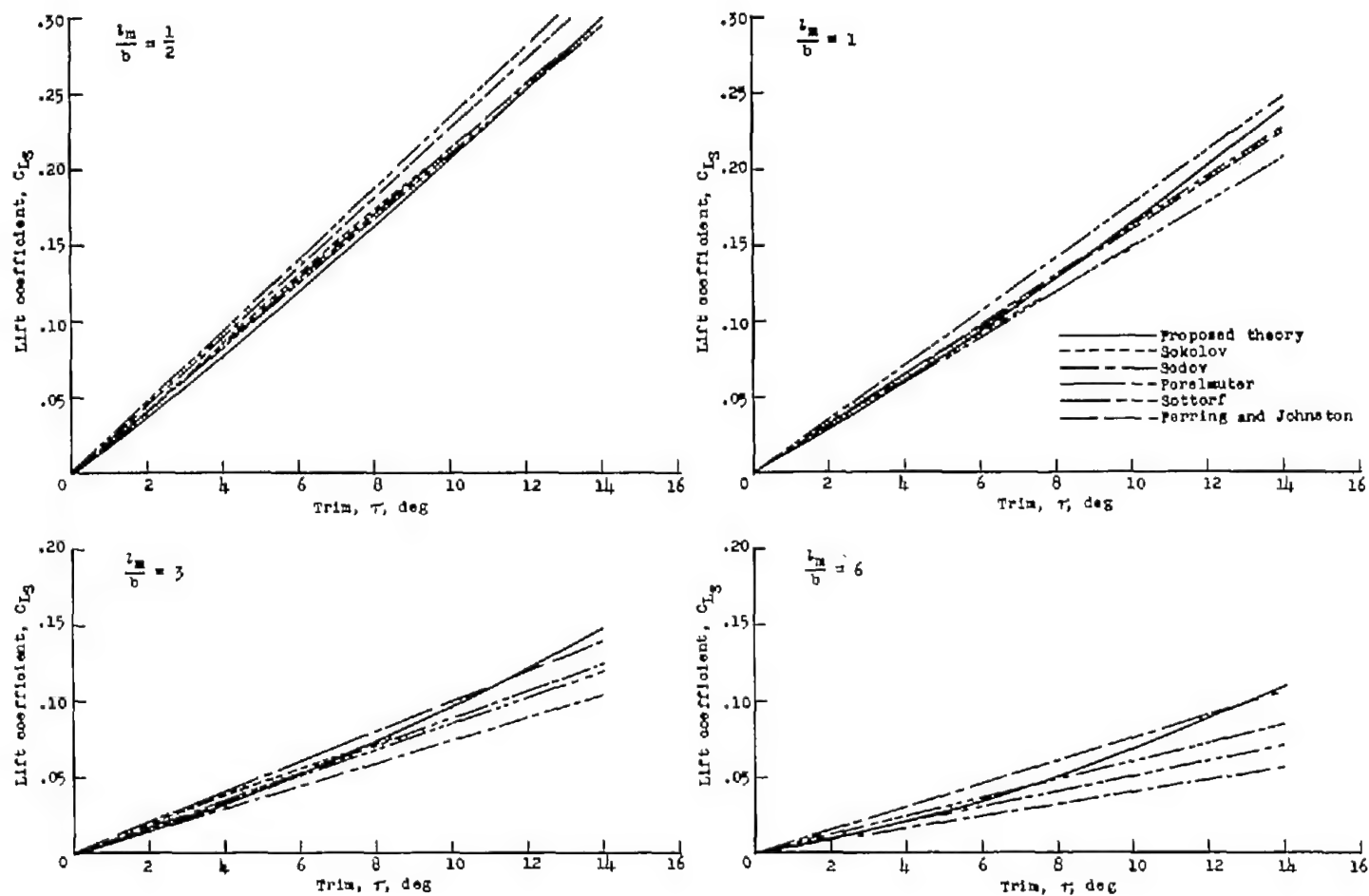
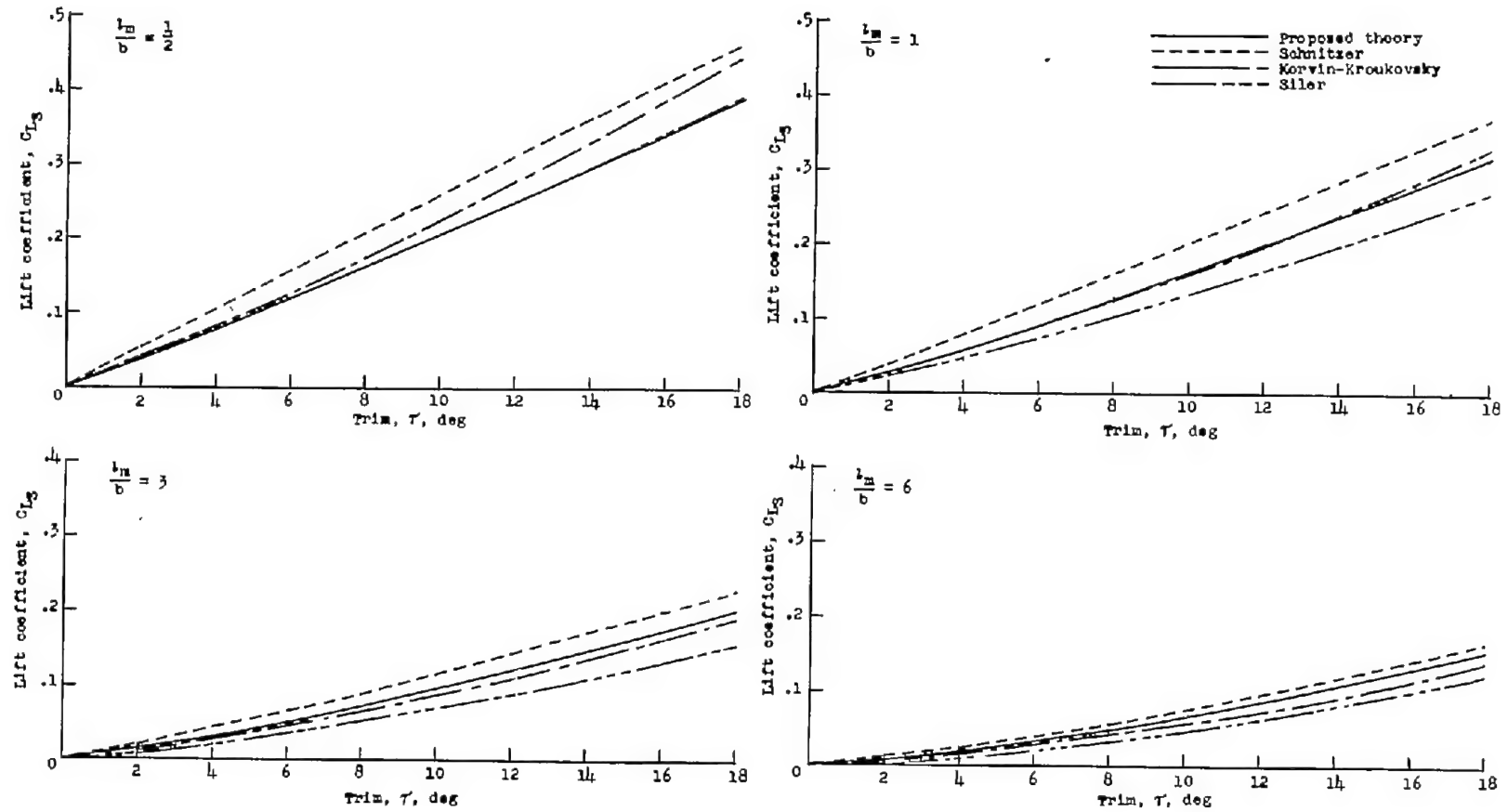


Figure 1.- Relative magnitude of components of proposed theory.



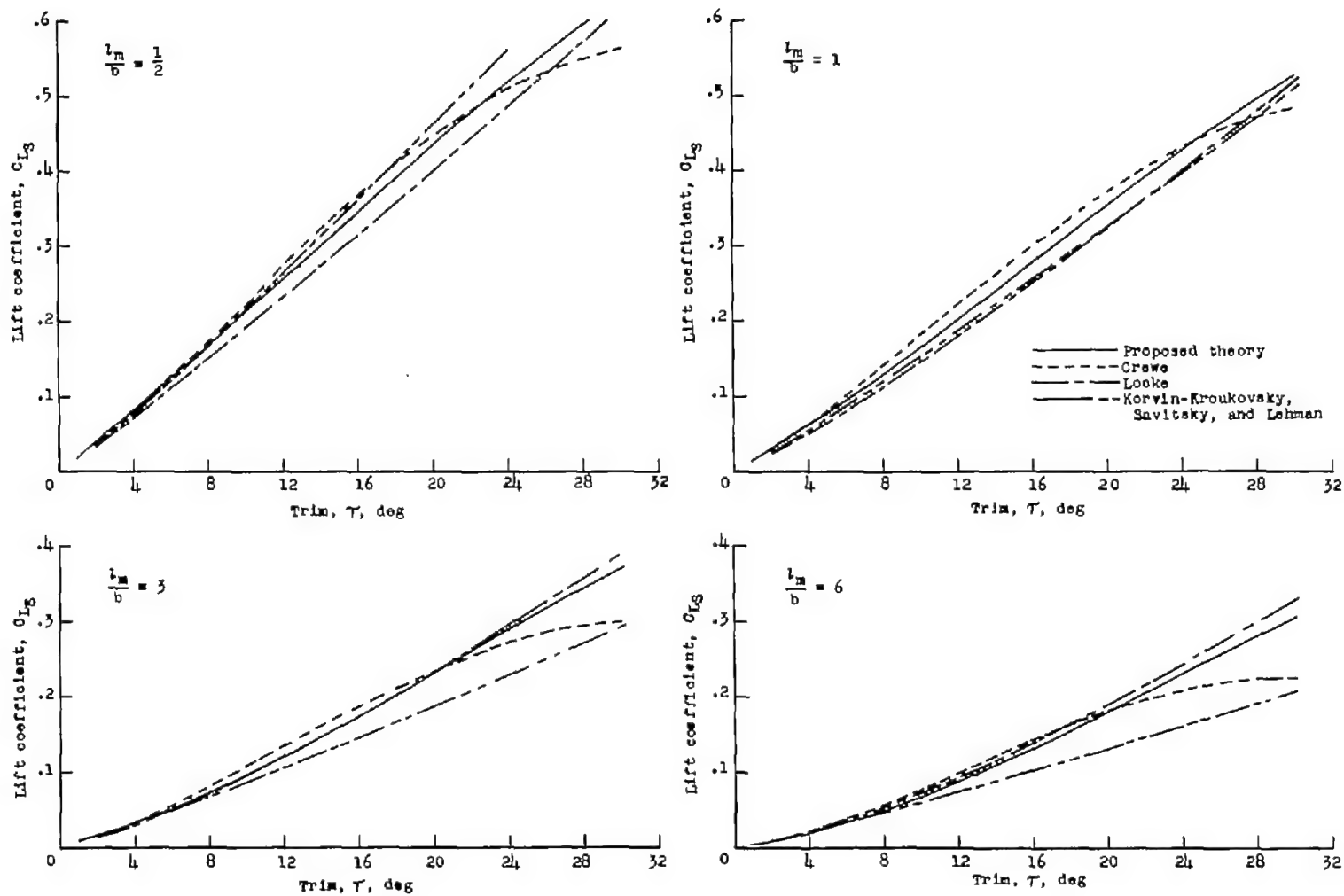
(a) Proposed theory and references 3 to 7.

Figure 2.- Variation of lift coefficient with angle of attack for rectangular-flat-plate lift formulas.



(b) Proposed theory and references 8, 10, and 13.

Figure 2.- Continued.



(c) Proposed theory, references 9 and 11, and Crewe's equation (eq. (12)).

Figure 2.- Concluded.

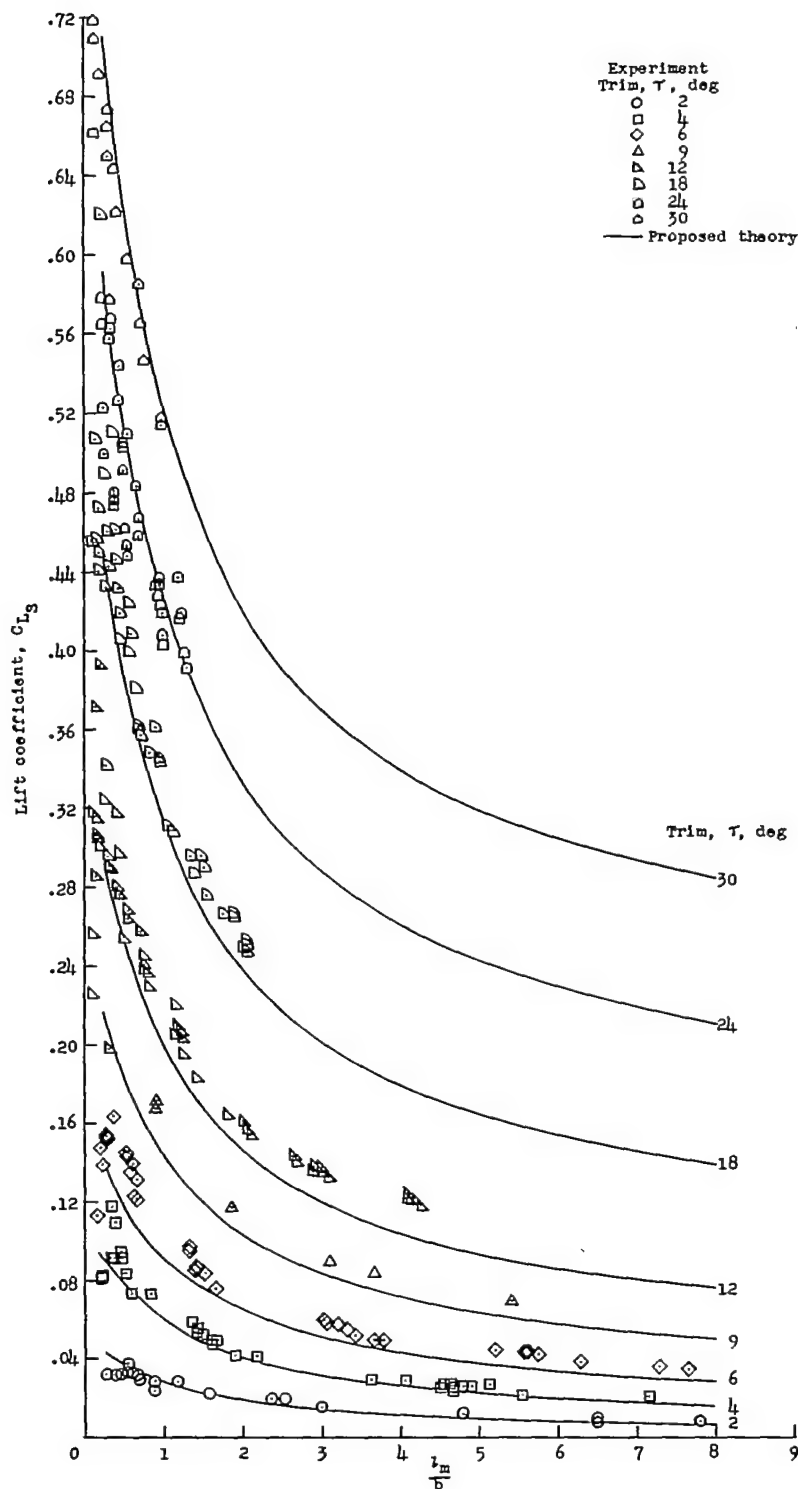


Figure 3.- Comparison of proposed theory with experimental lift coefficients for the 4-inch-beam rectangular-flat-plate planing-surface data of Weinstein and Kapryan (ref. 25).



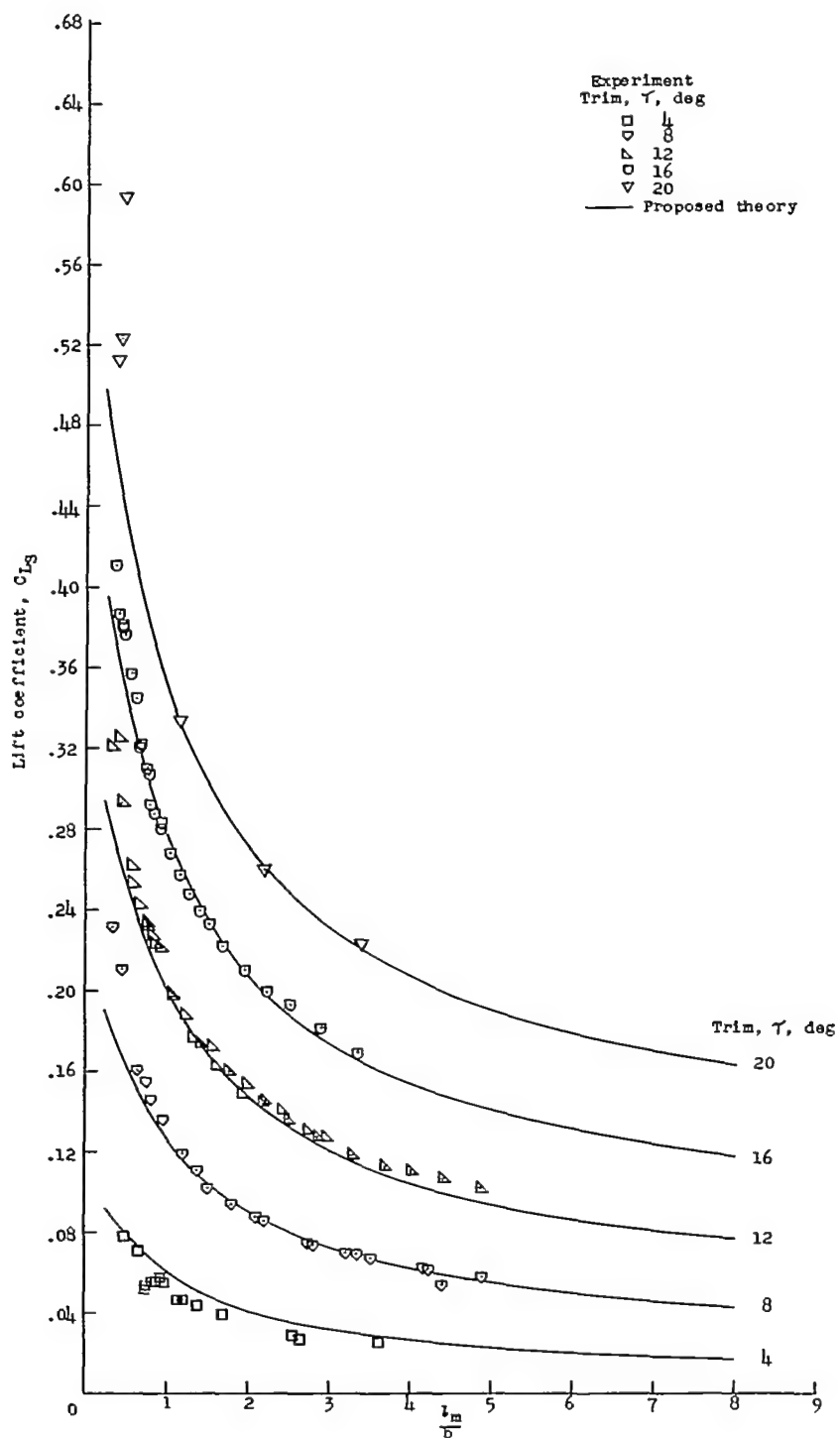
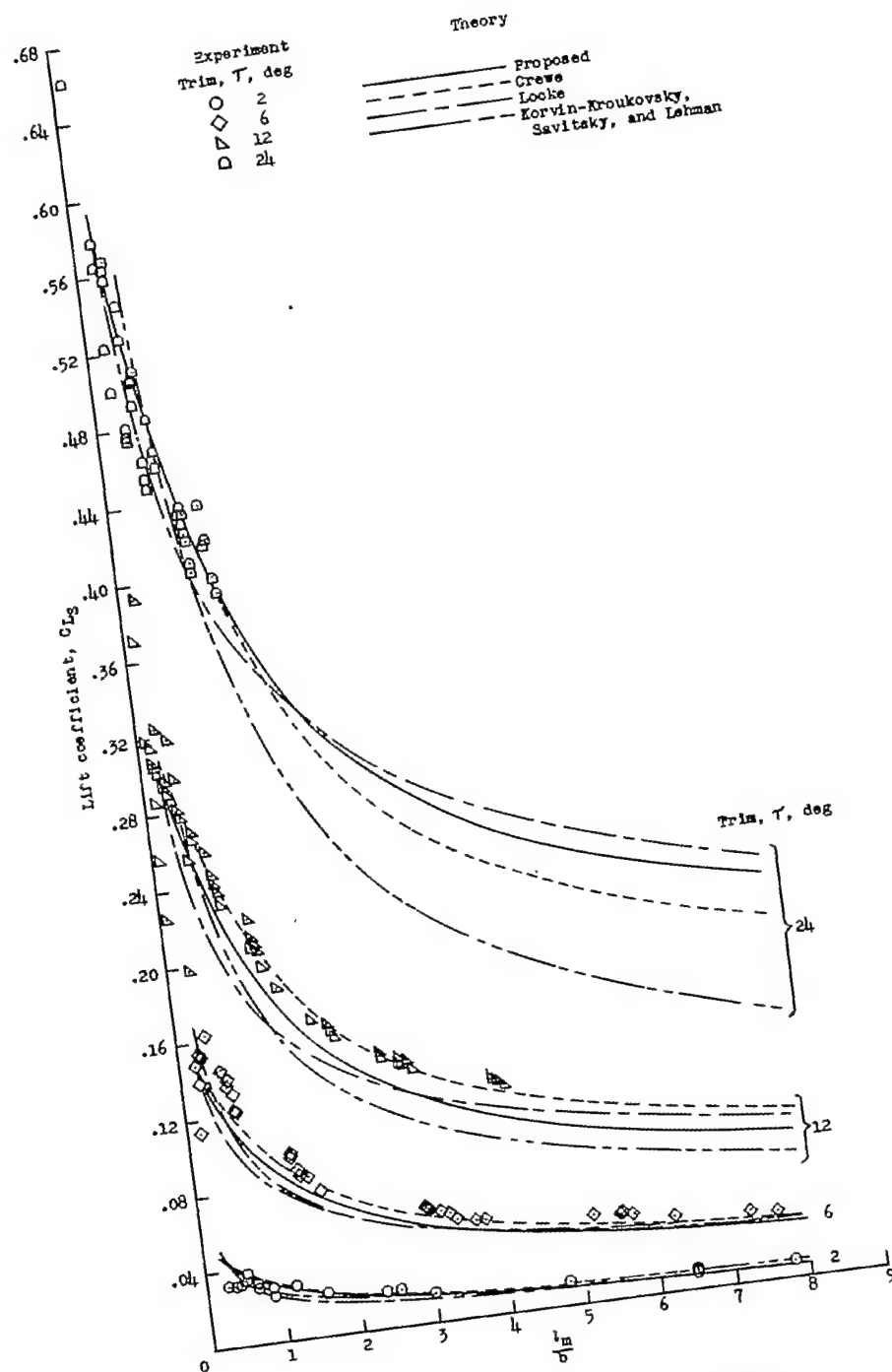
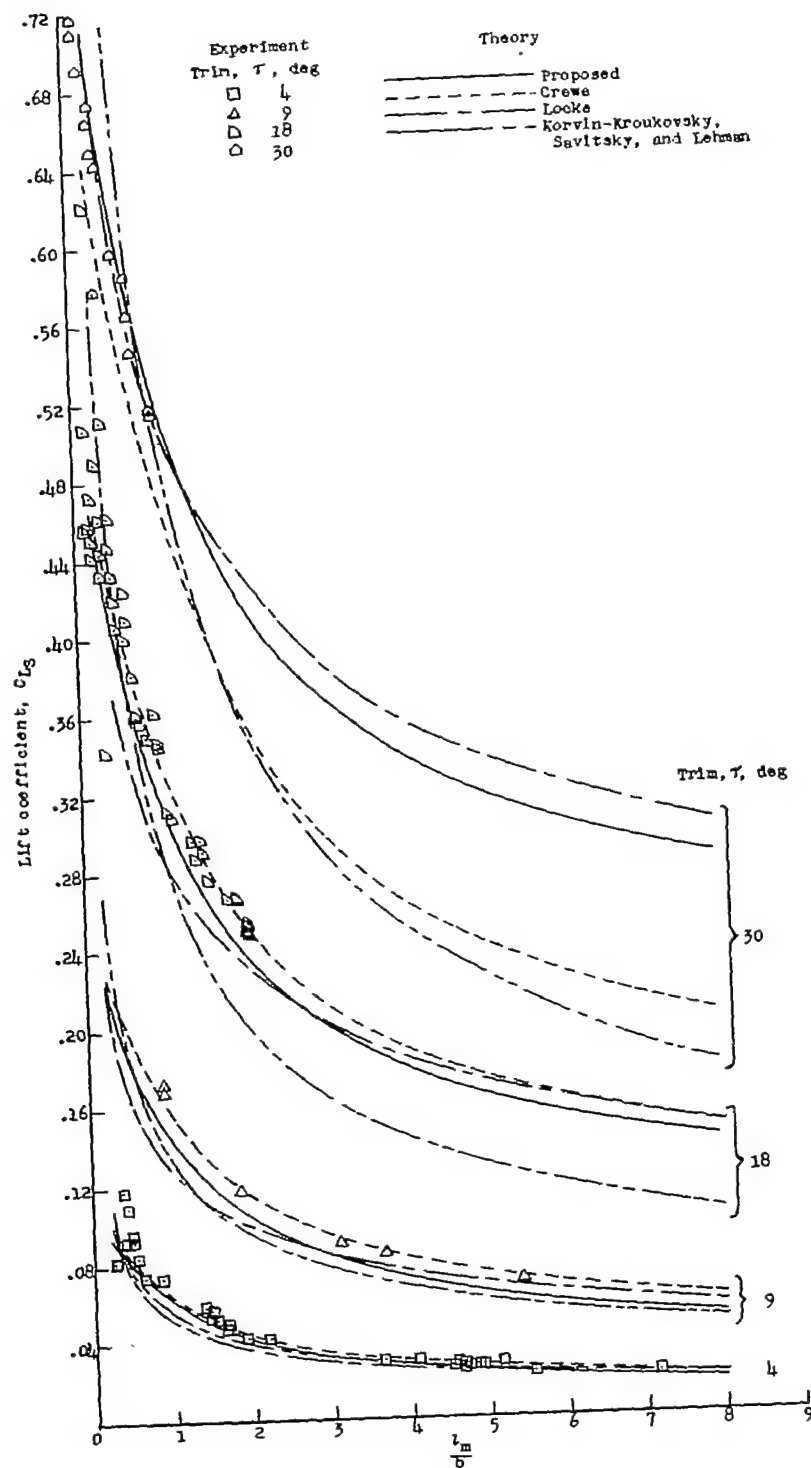


Figure 4.- Comparison of proposed theory with experimental lift coefficients for a 2.5-inch-beam rectangular-flat-plate planing surface (unpublished NACA data).



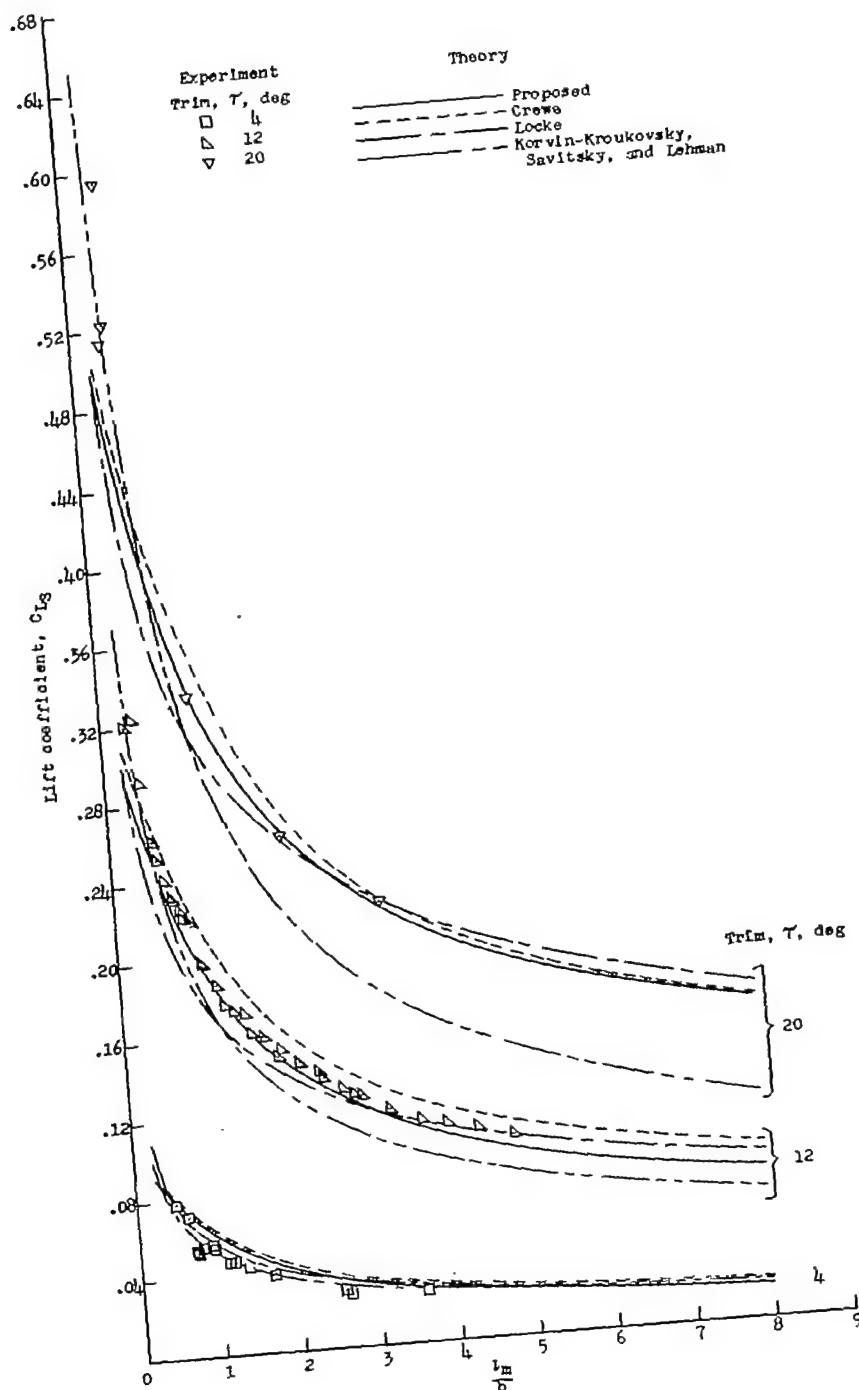
(a) Trim of 2°, 6°, 12°, and 24°.

Figure 5.- Comparison of proposed theory, formulas given by equations (8), (10), and (12), and experimental lift coefficients for the 4-inch-beam rectangular-flat-plate planing-surface data of Weinstein and Kapryan (ref. 25).



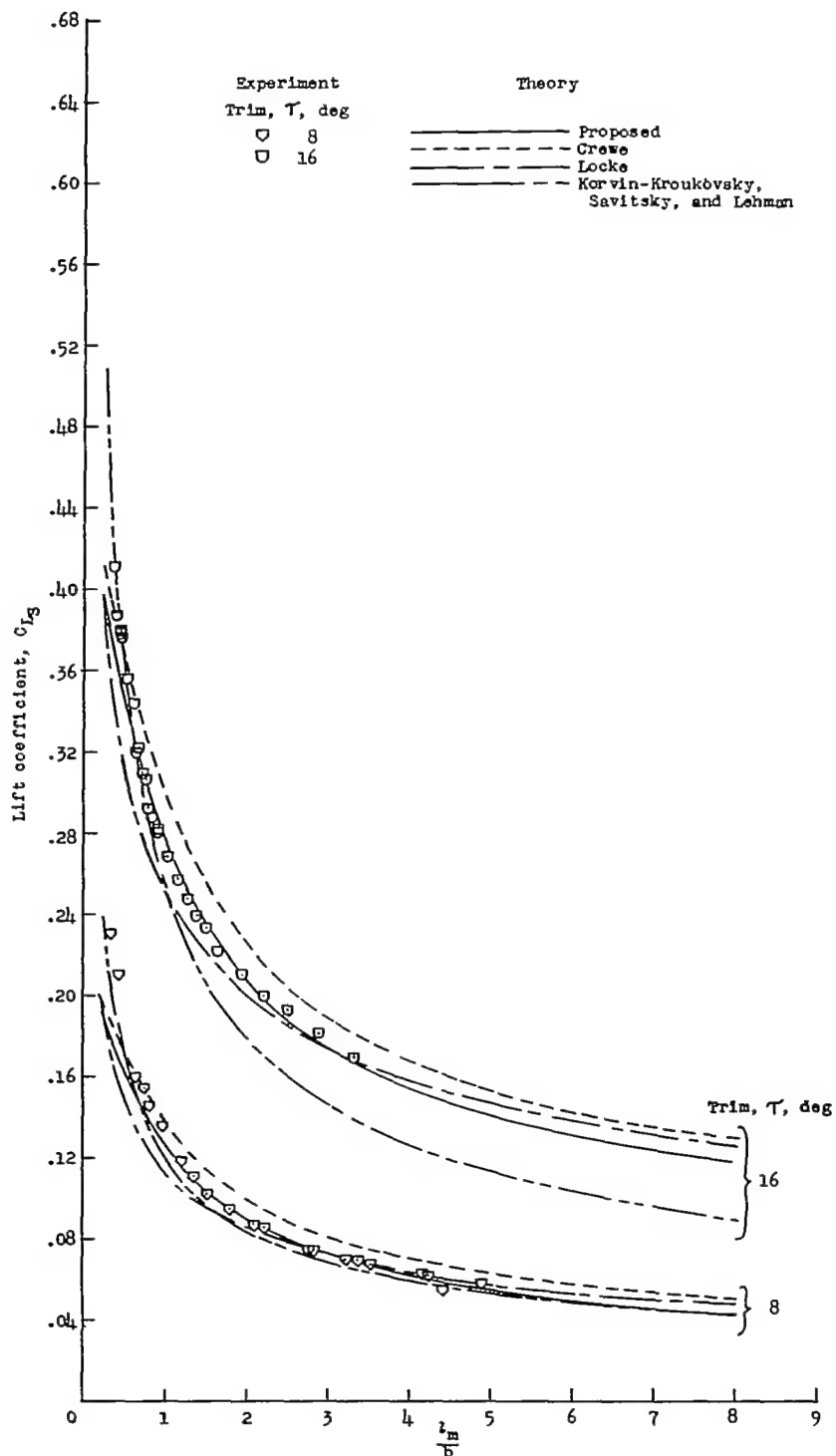
(b) Trim of 4°, 9°, 18°, and 30°.

Figure 5.- Concluded.



(a) Trim of 4°, 12°, and 20°.

Figure 6.- Comparison of proposed theory, formulas given by equations (8), (10), and (12), and experimental lift coefficients for a 2.5-inch-beam rectangular-flat-plate planing surface (unpublished NACA data).



(b) Trim of 8° and 16°.

Figure 6.- Concluded.

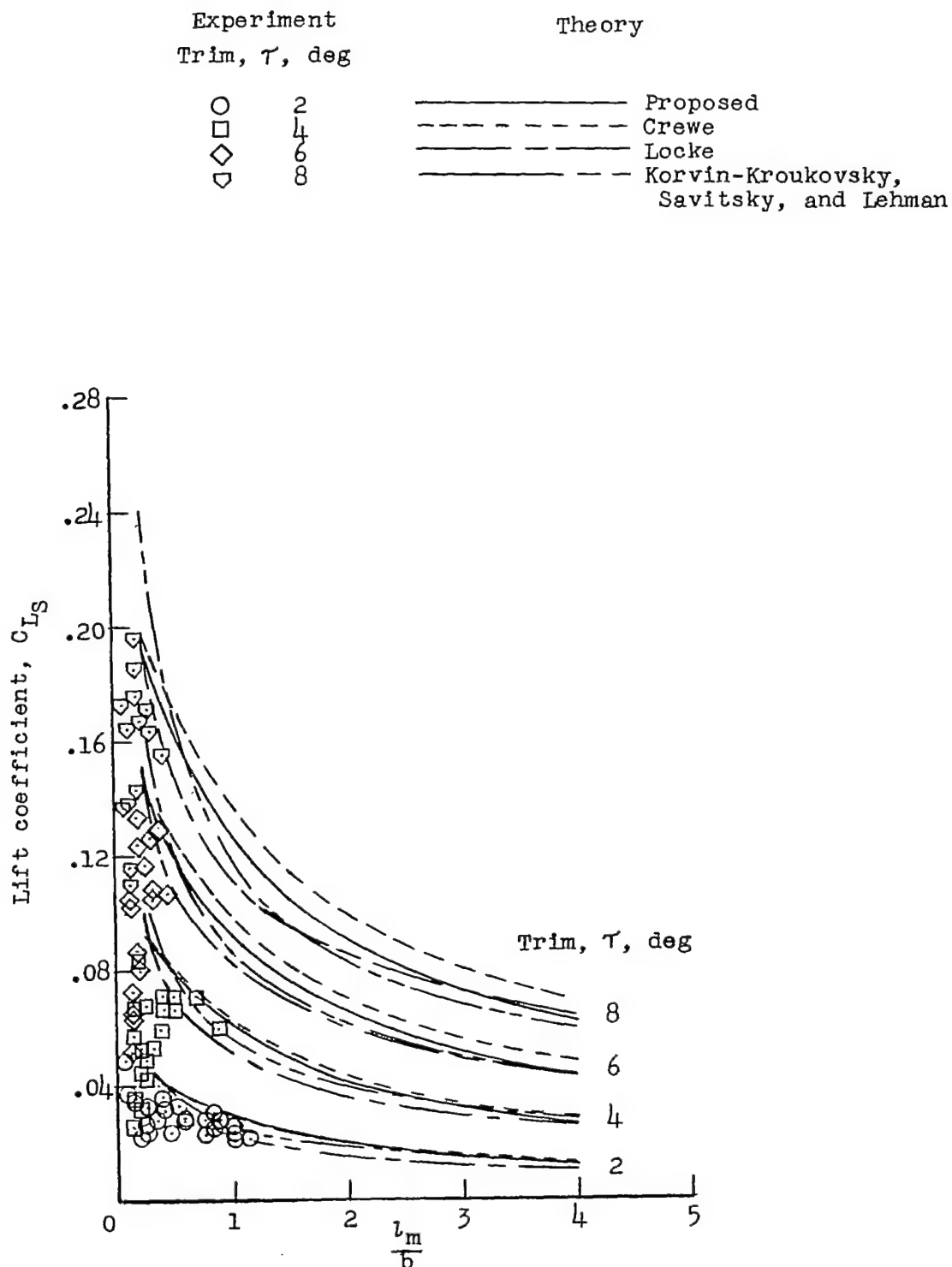


Figure 7.- Comparison of proposed theory, formulas given by equations (8), (10), and (12), and experimental lift coefficients for the 16-inch-beam rectangular-flat-plate planing-surface data of Shoemaker (ref. 26).

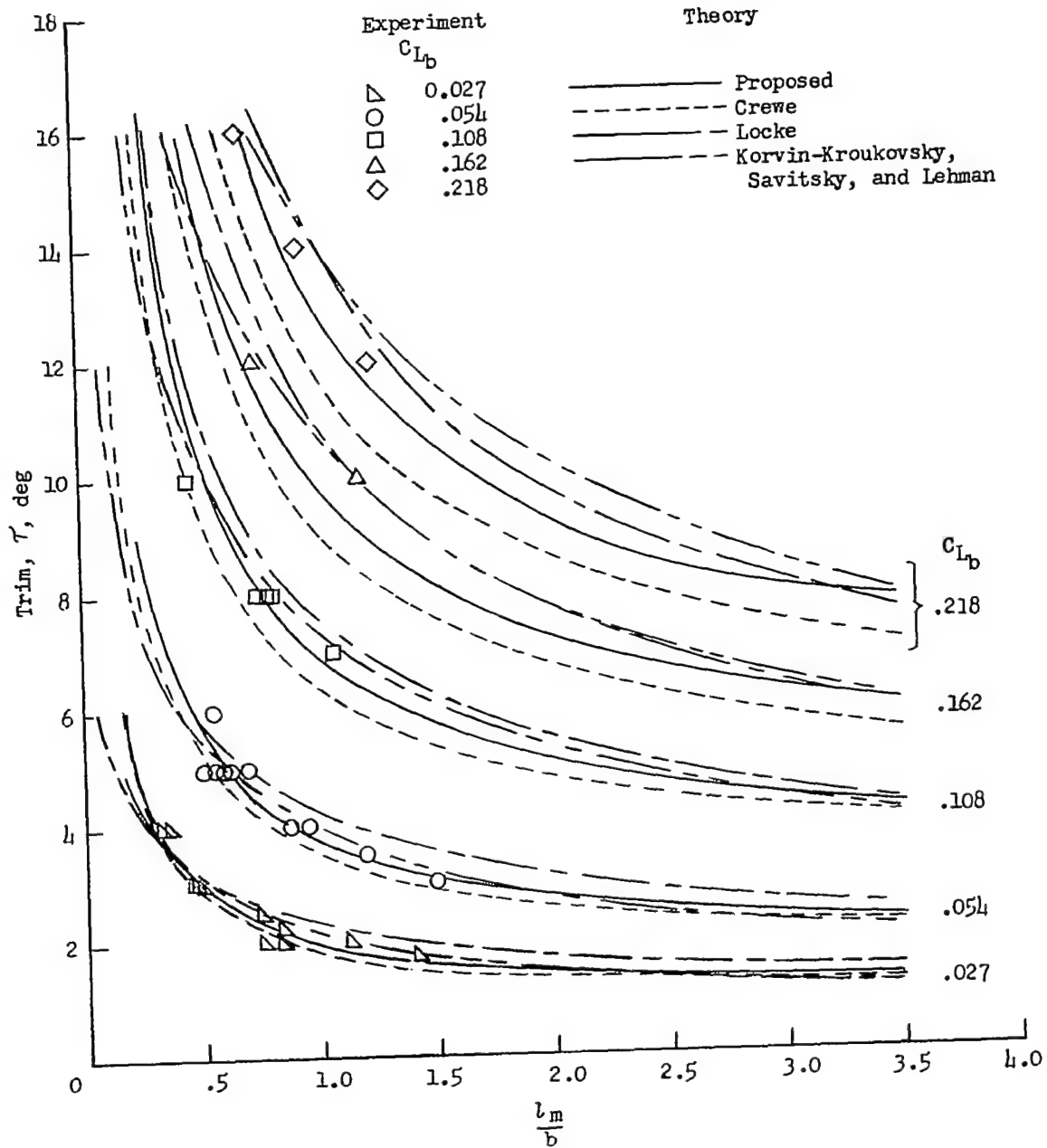


Figure 8.- Comparison of proposed theory, formulas given by equations (8), (10), and (12), and experimental lift coefficients for the 4-inch-beam rectangular-flat-plate planing-surface data of Locke (ref. 27).

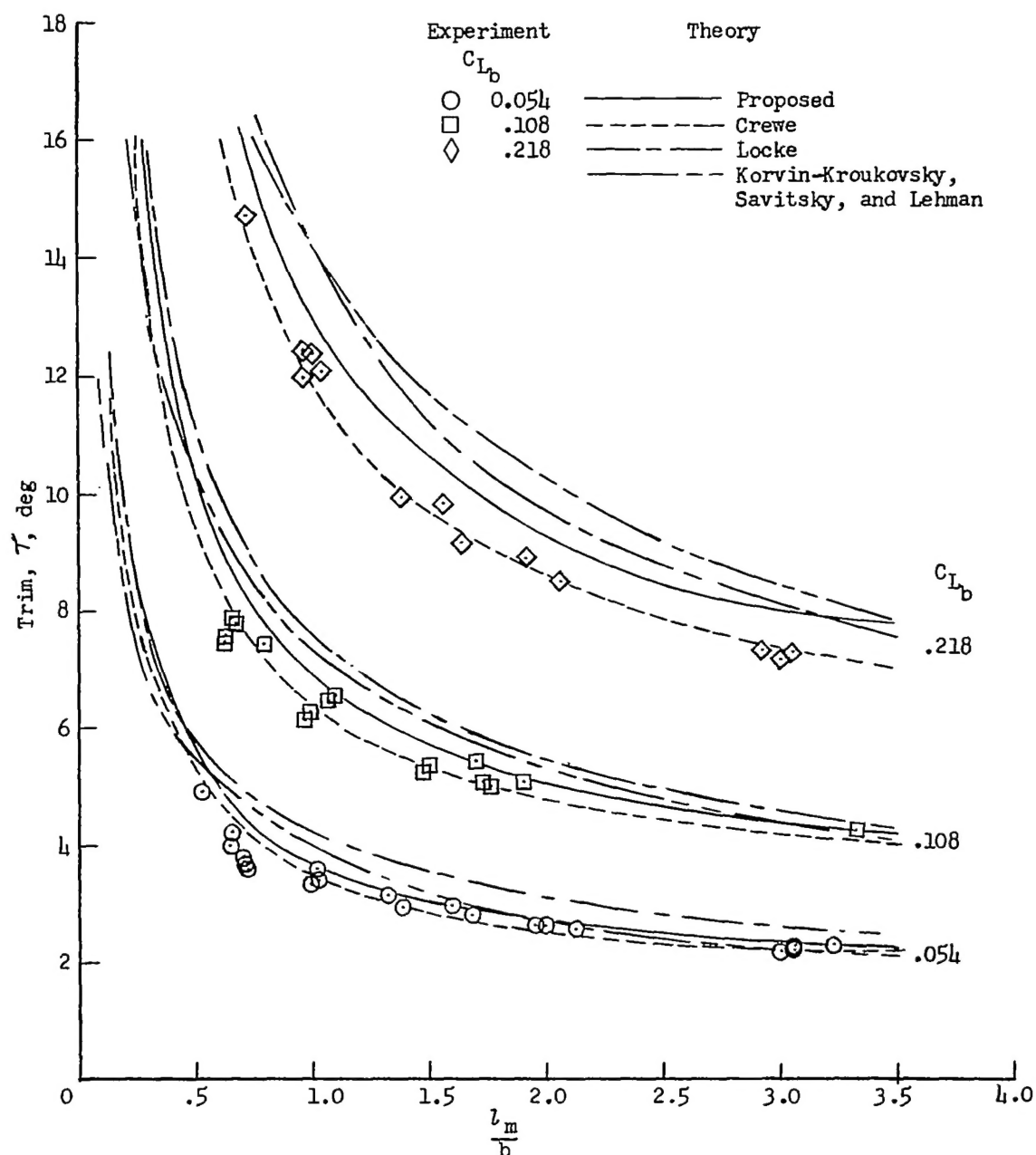


Figure 9.- Comparison of proposed theory, formulas given by equations (8), (10), and (12), and experimental lift coefficients for the 15- and 30-centimeter-beam (5.91- and 11.81-inch-beam) rectangular-planing-surface data of Sambraus (ref. 16).



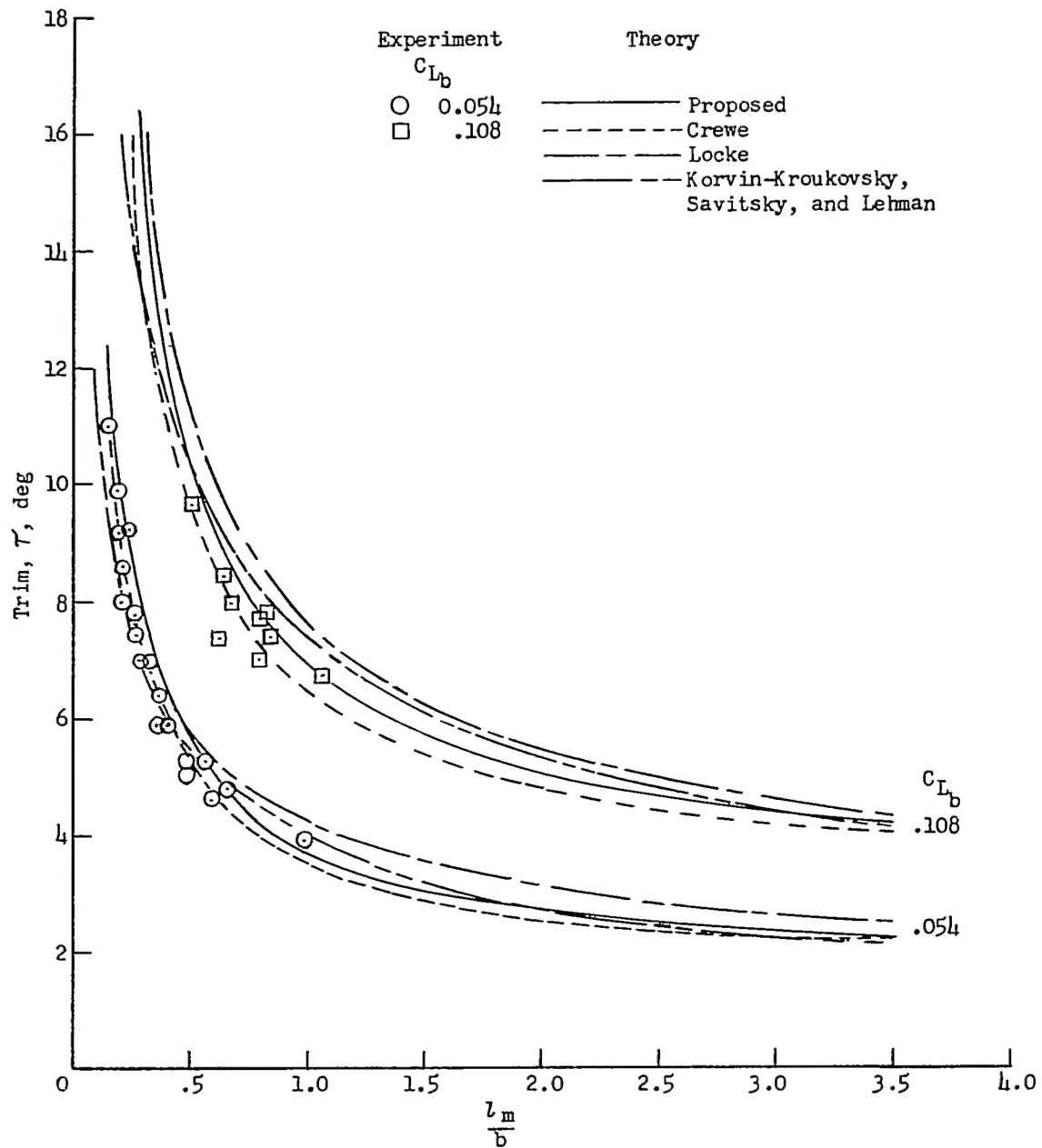


Figure 10.- Comparison of proposed theory, formulas given by equations (8), (10), and (12), and experimental lift coefficients for the 30-centimeter-beam (11.81-inch-beam) rectangular-planing-surface data of Sottorf (ref. 15).

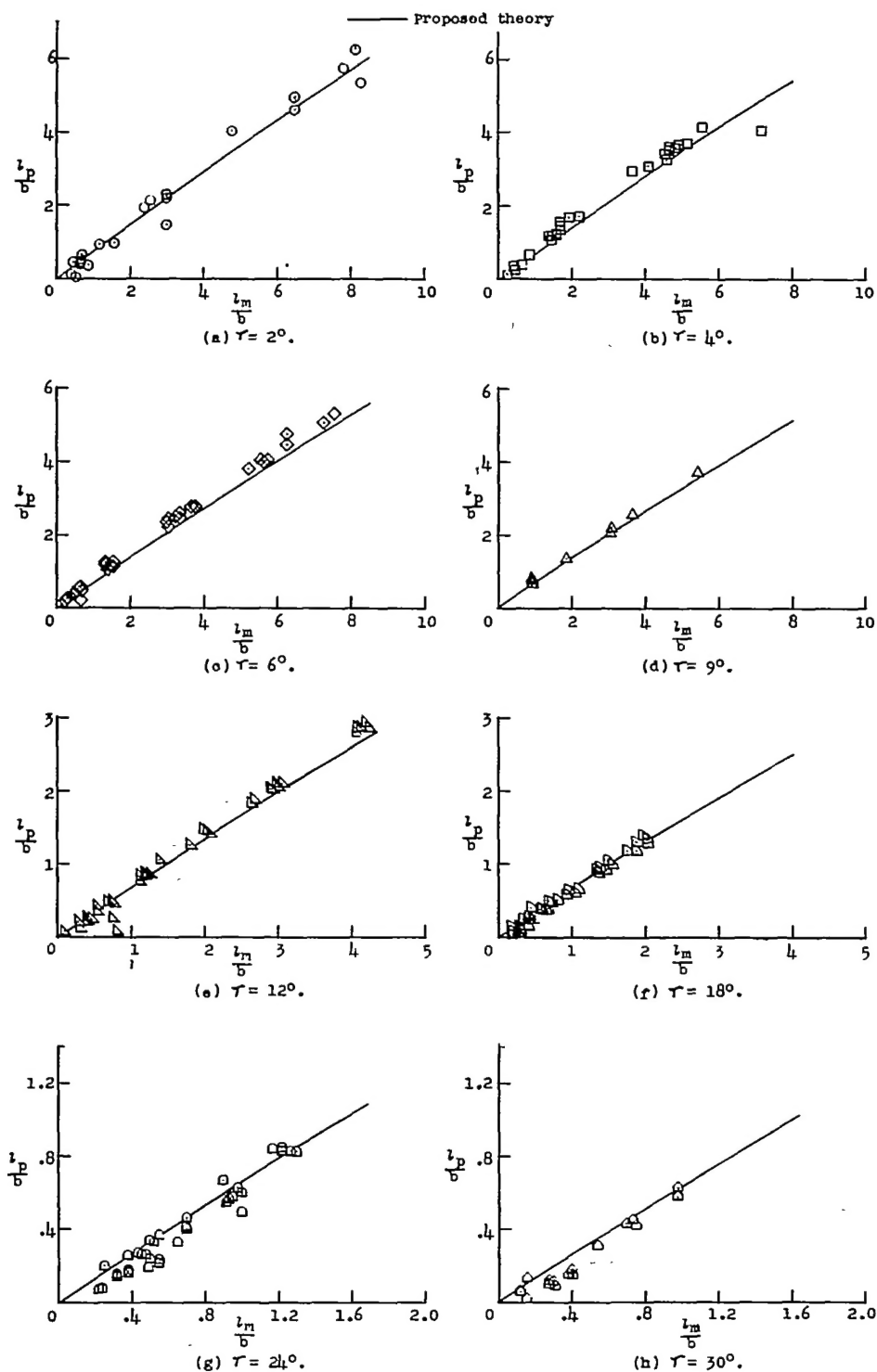


Figure 11.- Variation of center-of-pressure ratio with mean wetted-length-beam ratio for the 4-inch-beam rectangular-flat-plate planing-surface data of Weinstein and Kapryan (ref. 25).

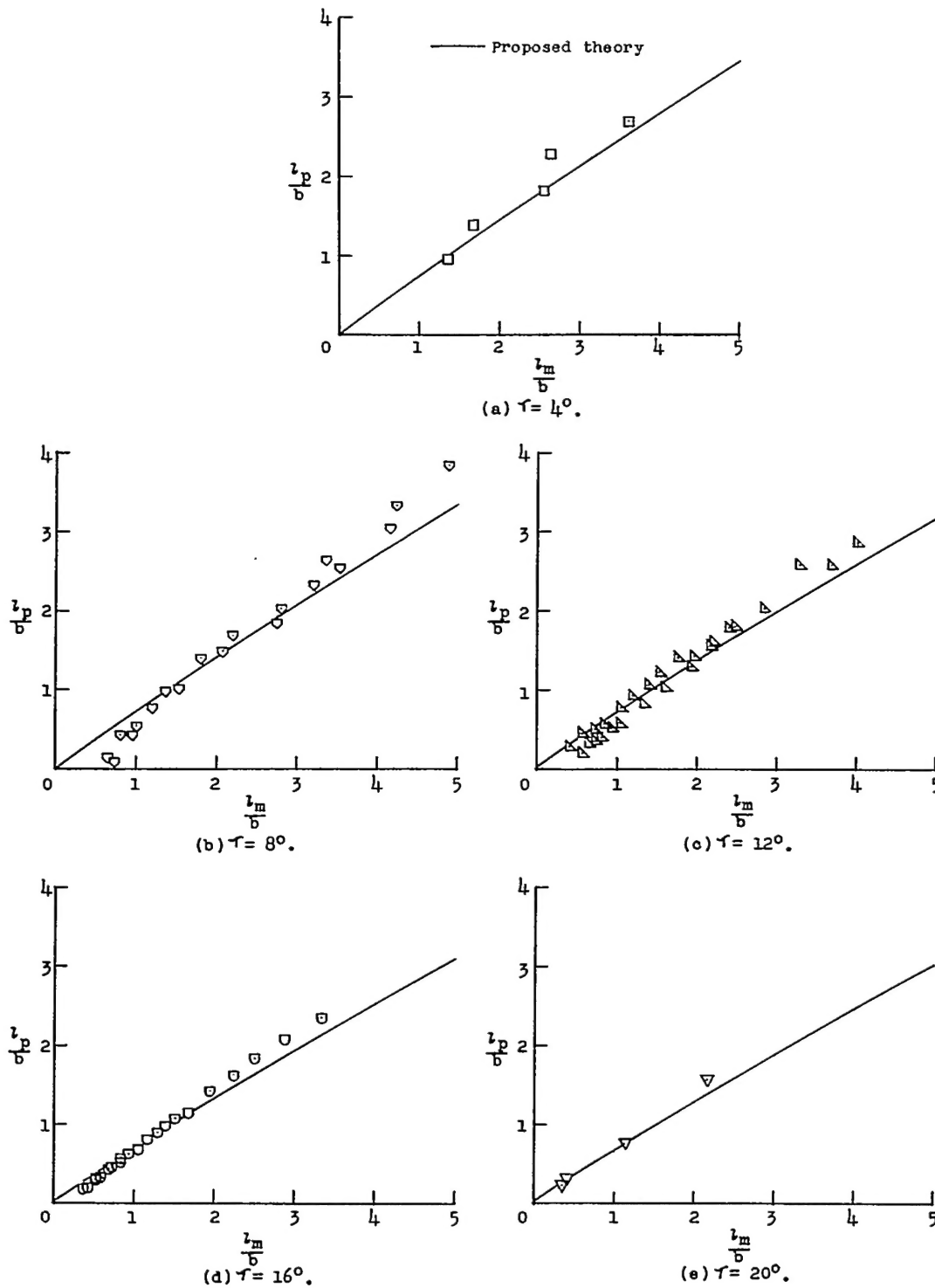


Figure 12.- Variation of center-of-pressure ratio with mean wetted-length-beam ratio for a 2.5-inch-beam rectangular-flat-plate planing surface (unpublished NACA data).